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A Modified Logistic Model to Incorporate Cyclical Fluctuations in Growth of *Tor putitora* (Hamilton)

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Abstract

Some growth models used in fisheries are inappropriate for use with species whose growth is seasonal due to the assumption that growth is invariant over time. A modified version of the logistic model that incorporates cyclical (or seasonal) fluctuations in growth gave significantly improved results when it was fitted to observations of length-at-age from *Tor putitora*. The modified logistic model was obtained by introducing a sine wave function into the original model.

Introduction

Tor putitora (golden mahseer) is an endangered coldwater fish species that is a popular fish as food and as a source of recreation for anglers. Singh et al. (2007) found that the von-Bertalanffy growth (VBG) model was suitable to explain the growth pattern of *Tor putitora* in polyculture and monoculture systems. However, the growth of fish in highly fluctuating or seasonal environments does not proceed at the same rate throughout

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the year. Thus, fish growth displays seasonal fluctuations, as was well known to the pioneers of fish biology (Fulton 1901). Consequently, seasonal growth models have been developed. For example, the VBG model was modified by addition of a sine wave function to adjust for seasonal variations (Pitcher and MacDonald 1973) and seasonally oscillating versions of the VBG model have also been developed by Somers (1988) and Hoenig and Hanumara (1990). It may not always be true that modified version of VBG model will give the best fit for all kinds of fish species when seasonal variation or cyclical fluctuation is involved. This paper aims to find a growth model that incorporates cyclical fluctuations in order to give a realistic representation of *Tor putitora* growth in the wild.

Materials and Methods

The most commonly used growth models in fisheries are:

Logistic growth model: $$L_t = L_\infty\left[1 + e^{-K(t-t_0)}\right]^{-1} \qquad (1)$$

Gompertz growth model: $$L_t = L_\infty \exp\left[-\exp\{-K(t-t_0)\}\right] \qquad (2)$$

Von-Bertalanffy growth model: $$L_t = L_\infty\left[1 - e^{-K(t-t_0)}\right] \qquad (3)$$

Richards growth model: $$L_t = L_\infty\left[1 + be^{-K(t-t_0)}\right]^{-1/b} \qquad (4)$$

Where,

L_t	Fish length at age t;
L_∞	Maximum fish length;
K	Growth coefficient (per year);
t	Age (in years);
t_0	Theoretical age (in years) when fish was length zero;
b	Added parameter in Richards model.

The von-Bertalanffy model with the sine wave function introduced by Pitcher and MacDonald (1973) is as follows:

$$L_t = L_\infty \left(1 - e^{-\left[C \sin\left(\frac{2\pi(t-S)}{52} \right) + K(t-t_0) \right]} \right) \qquad (5)$$

where C describes the magnitude of the growth oscillations around a non-seasonal or non-fluctuating growth curve, S is the starting point (relates to phase), and the 52 indicates a time scale in weeks.

In the similar fashion, when a sine wave is added to the logistic model (1), it becomes

$$L_t = L_\infty \left(1 + e^{-\left[C \sin\left\{ \frac{2\pi(t-S)}{P} \right\} + K(t-t_0) \right]} \right)^{-1} \qquad (6)$$

where P is the period of the cycle. Using P, rather than a fixed 52 weeks (as in equation (5)) extends the scope of the model.

Model Fitting

Equations (1) to (6) were applied to observations of length-at-age for *Tor putitora* collected in the foothill section of river Ganga and up-stream tributary Nayar during 1993-94 (see Bhatt et al. 2004 for further details).

The growth models are non-linear, details of non-linear models having been given by Ratkowsky (1990). There are four main methods (Seber and Wild 1989) for obtaining estimates of the unknown parameters in non-linear regression models: Gauss-Newton Method, Steepest-Descent Method, Levenberg-Marquardt Technique and Do Not Use Derivative (DUD) Method. The Levenberg-Marquardt method is the most widely used and reliable procedure for computing non-linear least square estimates and was used in the present study.

To examine model performance, a measure of how the predicted and observed variables covary in time is needed. For non-linear models, Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) are often used for assessment:

$$\text{RMSE} = \left[\sum_{t=1}^{n} \left(L_t - \hat{L}_t \right)^2 / n \right]^{1/2} \text{ and } \text{MAE} = \sum_{t=1}^{n} \left| \left(L_t - \hat{L}_t \right) \right| / n,$$

where

$\hat{L}_t$ Predicted fish length at age t;

n Number of observations, $t = 1,2,...,n$.

The best model will have the lowest numerical values for these statistics. In addition, independence or the randomness assumption of the residuals needs to be tested before taking final decision about the adequacy of the model. To test the independence assumption of residuals the run test procedure is available (Ratkowsky 1990), but the normality assumption is not stringent for selection of non-linear models because their residuals may not follow normal distribution.

The models were fitted using the Non-linear Regression option on SPSS 12.0 version. Different sets of initial parameter values were tried to meet the global convergence criterion for best fitting of the non-linear models.

Results and Discussion

The von-Bertalanffy and Richards growth models failed to give optimal solutions whereas the logistic and Gompertz models gave optimal solutions. The estimates of parameters, RMSE, MAE, run test statistic (|Z|) value are presented in table 1. The Gompertz model performs better than the logistic model when RMSE and MAE criteria are used to identify the best model (Table 1). The, independence assumption about residuals is satisfied because run test |Z| values are below the critical value (1.96 at 5% level of significance). When residuals of the Gompertz and logistic models were fitted against expected length (Figure 1), a cyclical pattern was seen. Addition of a sine wave to the model is suggested to give a solution. The modified versions of the Gompertz and von-Bertalanffy models failed to meet global convergence, whereas the logistic model with sine wave gave the optimal solution. The parameter estimates are given in table 1. The values of RMSE and MAE were improved, as compared to the simple Gompertz and logistic models, and the run test $(|Z|)$ value is 0.76 (less than the critical value 1.96). Model growth predictions depicted in figure 2 along with observed values. The modified version of the logistic model describes the *Tor putitora* data better than other popular growth models. Moreover, the asymptotic length of *Tor putitora*, estimated using the modified logistic model is approximately 189 cm (Table 1): this seems acceptable because the maximum size recorded in India is 275 cm (Jhingran 1975) and the largest size in Nepal is 180 cm (Shrestha 1999).

Table 1. Summary statistics for fitting of various non-linear models

	Logistic	Gompertz	Logistic with sine wave
(1) Parameter estimates			
L_∞	206.37 (79.08)*	545.58 (517.80)	188.54 (44.51)
K	0.25 (0.04)	0.08 (0.03)	0.27 (0.02)
t_0	10.01 (2.82)	16.44 (9.74)	9.19 (1.52)
S	-	-	9.42 (0.85)
C	-	-	0.10 (0.02)
P	-	-	6.56 (1.42)
(2) Model adequacy			
RMSE	2.28	2.06	0.65
MAE	2.15	1.96	0.48
(3) Residual analysis			
Run test $(\lvert Z \rvert)$	0.68	0.68	0.76

*Bracketed values are the corresponding asymptotic standard errors.

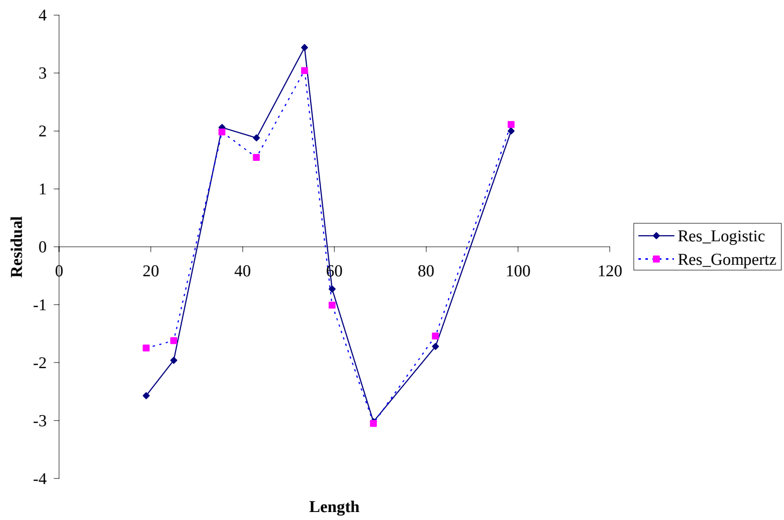

Figure 1. The residuals remaining after fitting of Gompertz and logistic models to the data

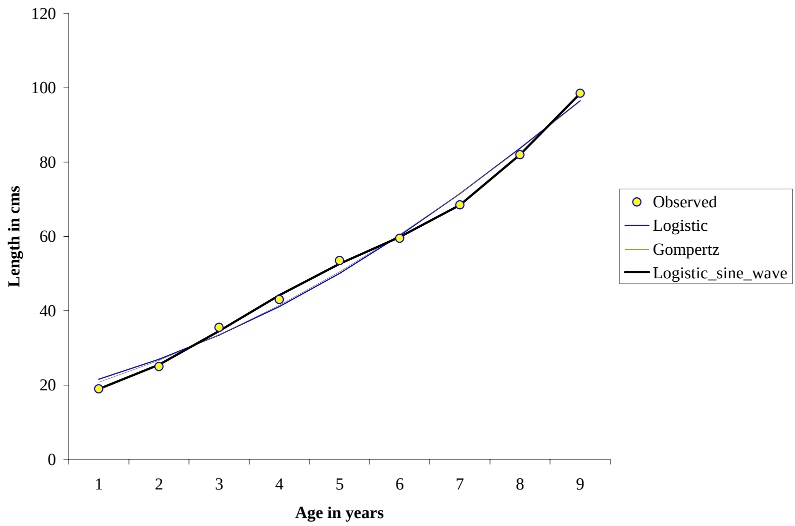

Figure 2. Graphical display of measured and predicted growth in length of *Tor putitora*

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